Exercise sheet: Due April 25

1. Show the hyperbolic cosine rule for the triangle T: if $\rho(0, v_a) = b$, $\rho(0, v_b) = a$, $\rho(v_a, v_b) = c$ and the angle at the origin is γ, then

$$\cosh 2c = \cosh 2a \cosh 2b - \sinh 2a \sinh 2b \cos \gamma$$

2. Show the hyperbolic sine rule in the triangle T: if α is the angle at vertex v_a and β is the angle at vertex v_b, then

$$\frac{\sinh 2a}{\sin \alpha} = \frac{\sinh 2b}{\sin \beta} = \frac{\sinh 2c}{\sin \gamma}$$

3. Let P be the ideal hyperbolic rectangle in the unit disk with vertices at $(e^{i\theta}, -e^{-i\theta}, -e^{i\theta}, e^{-i\theta})$. Let d be the hyperbolic length of the geodesic between the points where its sides meet the real axis and let a be hyperbolic length of the geodesic between the points where its sides meet the imaginary axis. Show that

$$\sinh a \sinh d = 1 \tag{1}$$

4. Find the hyperbolic density for the punctured unit disk.

5. Show that if A and B are hyperbolic transformations whose axes are disjoint and not tangent at the boundary, then there is a common orthogonal to these axes.