

Exercises due 11/1

Ahlfors (p.119 in 1953 edition)

- (1) Prove that the region obtained from a simply connected region by removing m points has connectivity $m + 1$ and find a homology basis.
- (2) Show that a single valued analytic branch of $\log z$, z^α , and z^z can be defined in any simply connected region which does not contain the origin.
- (3) Show that a single valued analytic branch of $\sqrt{1 - z^2}$ can be defined in any region such that the points ± 1 are in the same component of the complement. What are the possible values of

$$\int \frac{dz}{\sqrt{1 - z^2}}$$

over a closed curve in the region?