Exercises due 10/25

(1) Prove that a function analytic in the whole plane and satisfying an inequality of the form $|f(z)| \leq |z|^n$ for some n and sufficiently large $|z|$ must reduce to a polynomial.

(2) If $f(z)$ is analytic and $|f(z)| \leq M$ for $|z| \leq R$, find an upper bound for $|f^{(n)}(z)|$ in $|z| \leq \rho < R$.

(3) If $f(z)$ is analytic for $|z| < 1$ and $|f(z)| \leq 1/(1 - |z|)$, find the best estimate of $|f^{(n)}(0)$ that Cauchy's inequality will yield.