Midterm Exam Complex Analysis Fall 2008

Answer eight of the eleven questions. Each is worth 12.5 points.

1. Find all numbers a such that $a^3 = -1$.

2. Determine where the following functions are continuous and where they are analytic:

$$f(z) = xy + iy \qquad f(z) = \sin x \cosh y + i \cos x \sinh y$$

3. If a rational function is real on $|z| = 1$ how are its zeros and poles situated?

4. Show the following function is harmonic a find a conjugate harmonic function: $u(x, y) = \cosh x \cos y$

5. For what values is

$$\sum \left(\frac{z}{1+z}\right)^n$$

convergent?

6. Show that an analytic function cannot have constant argument unless it is constant.

7. Show that the union of two disks is connected if and only if the distance between the centers is less than the sum of the radii.

8. If the radius of convergence of $\sum_n a_n z^n$ is R_a and if the radius of convergence of $\sum_n b_n z^n$ is R_b prove that the radius of convergence of $\sum_n a_n b_n z^n$ is at least $R_a R_b$.

9. Find a conformal map from the region bounded by the circles $|z| = 1$ and $|z - 2| = 3$ to the upper half plane.

10. Prove that a function analytic in the whole plane and satisfies $|f(z)| < |z|^n$ for some n and sufficiently large $|z|$ must be a polynomial.

11. Find $\int_{\partial R} \frac{dz}{z^2+4}$ where R is the square of side 6 centered at the origin and the integral is taken around the boundary in the counterclockwise direction.