Agreement with stopping failures

Problem definition:
n-node connected undirected graph
inputs, outputs in V
synchronous model
<= f failures; all failures are stopping failures (ie, the process stops, it doesn't misbehave)

Agreement: no 2 processes that decide, decide on unequal values
Validity: if all start with v then v is only possible decision
Termination: all nonfaulty processes eventually decide

Complexity measures
Time: rounds until all nonfaulty decide
Communication: number messages

Algorithms

Maintain a set $W \subseteq V$ of all values you've seen, initially your own initial value
Repeatedly broadcast W
After k rounds, if W contains a single value, pick it; otherwise, pick default v_0.
Alternatively, pick the smallest value in W – this requires a totally ordered set V.

How many rounds? Let's consider the number of failures:

1. No failures: 1 round is enough.
2. 1 failure: If the process with the only different value fails in the middle of sending, then some processes have two values and some have only one. Hence, need 2 rounds.

In general: f+1 rounds

Theorem: After f+1 rounds, all processes pick the same value and if all processes started with the same value they all pick it.
Proof Sketch:

1) If no process fails during a round, then all processes have the same set W, ie, all have seen the same set of values.
2) Once the W's are equal, they stay equal.
3) if the W's are equal, all processes decide the same.
4) There is at least one round in the first f+1 rounds when no process fails.

Agreement: See claims 2) and 3)
Validity: If all start the same, see 2) and 3)
Termination: ?

Complexity: f+1 rounds, $O((f+1)*n^2)$ messages.

Optimization:
Broadcast only the first 2 values – the decision is based on the size of W for the default decision rule.
For proof, use simulation relation.

Lower bound on number of rounds: f+1

Can prove by assuming we have an f round algorithm and getting a contradiction.