CMACS
Computational Modeling and Analysis for Complex Systems

# 2010 CMACS Workshop on Modeling Biological Systems

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# Temporal Logic

## Outline

- *Propositional Logic*
- *Linear Temporal Logic (LTL)*
- *Branching Temporal Logic (CTL)*
- *Formal Verification*

## Logic

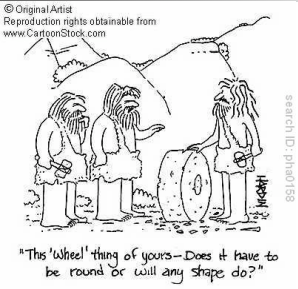
What is logic?

Wikipedia:

*Logic, from the Greek λογικός (logikos) is the study of reasoning...*

*Logic is used in most intellectual activity, but is studied primarily in the disciplines of philosophy, mathematics, and computer science.*

## Logic




## Propositional logic

Propositional logic is the simplest of all logics.

## Propositions

It is two o'clock.
The mind is the brain.
The mind is not the brain.
He believes in God.

Where have you been?
Oh no, not logic again.
Thank you.
Help!

## Propositions

It is two o'clock.
The mind is the brain.
The mind is not the brain.
He believes in God.

Propositions:
- Things we can believe
- Things which can be true or false
- Things expressed by declarative sentences

## Propositional Logic

- *Proposition letters: p, q, r, s etc.*
- *Logical connectives:* $\wedge$ and
  $\vee$ or
  $\neg$ not
  $\rightarrow$ implies
- *Syntax:*
  $\varphi := p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi$

## Propositional Logic

- *Proposition letters: p, q, r, s etc.*
  - *p* = It's raining.
  - q = Everything is determined.
  - r = People are free.
  - s = It's snowing

- *Sentences:*

It is not true that it is raining: ¬p

Everything is determined and it is snowing: q ∧ s

If it's raining then people are free: p → r

People are free or everything is determined: r ∨ q

It is not true that people are free and everything is determined: ¬(r ∧ q)

## Propositional Logic: exercises

Translate the following sentences in propositional logic:

- If The Netherlands will win the world cup (w), then pigs will fly (p).
- Pigs will not fly, and bacon will be free (q).
- The Netherlands will win the world cup, or bacon will be free, but not both.

## Temporal logic

Temporal logic: a notation for the representation of temporal information.

I will be happy.

She will study until she passes the exam.

He did not do the dishes, but he will have to at some point.

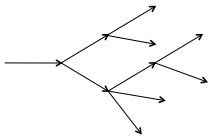
## Time

What does time look like?

Is it linear?



Or, is it branching?



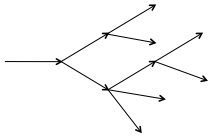
## Time

What does time look like?

Is it linear? Linear Temporal Logic (LTL)



Or, is it branching? Branching Temporal Logic (CTL)

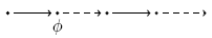
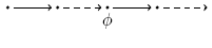
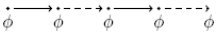
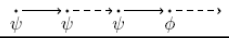


## Linear Temporal Logic (LTL)

*Syntax:*

ϕ:= p| ¬ ϕ| ϕ ∨ψ | ϕ ∧ψ | Xϕ | Fϕ| Gϕ| ϕ U ψ

*Temporal Operators:*

- Xϕ ne**X**t 
- Fϕ **F**inally 
- Gϕ **G**lobally 
- ψ U ϕ **U**ntil 

## Linear Temporal Logic (LTL)

Some LTL fomulas:

- I am always hungry
  G h
- I will eventually be hungry
  F h
- I will be hungry until I eat something
  h U e
- In the future, I will eat something, and then I will never be hungry again
  F(e ∧ G ¬ h)

## LTL: Exercises

Let p = 'I drink something' and t = 'I am thirsty'. Write in LTL

1. *I will drink something in the future.*
2. *I am thirsty until I drink something.*
3. *I will drink something and then I will not be thirsty anymore.*
4. *There will always be a future moment in which I am thirsty.*
5. *There will always be a future moment in which I am thirsty, and then I will eventually drink something.*
6. *I will always be thirsty* (Without using G)
7. ** I am thirsty, and If I ever drink something, I will not be thirsty again.*
8. * Do 1. without using F

## LTL: Exercises

1. *I will be thirsty over and over again, forever.*
2. *I will drink something over and over again, forever.*
3. *I will be thirsty until I drink repeatedly, forever.*

## Adding relational statements to LTL

We can add constants (0,1 ),
variables (x, y, z) ,
and relation symbols (≤, ≥ )
to our language.

- Some time in the future, the x will be smaller than 1
  F(x < 1)
- The value of x never drops below 0
  G(x ≥ 0)

## More exercises:

Write in LTL:

1. The value of x will always be between 0 and 1.
2. It will happen infinitely often that x is greater than 1, and that x is below 0.

## Branching Temporal Logic (CTL)

*Syntax:*

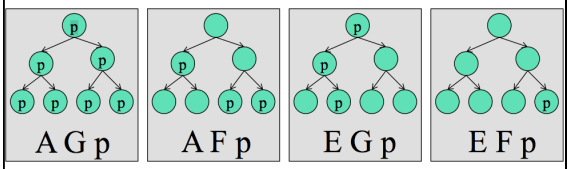
$\phi := p \mid \neg\phi \mid \phi \vee \psi \mid \phi \wedge \psi \mid X\phi \mid F\phi \mid G\phi \mid \phi\, U\, \psi \mid E\, \mathbf{Z}\, \phi \mid A\, \mathbf{Z}\, \phi$

*Where* ***Z*** *= X, F, G, U.*

*Two path quantifiers:*
*E = for some path*
*A = for all paths*

## Branching Temporal Logic (CTL)

*Two path quantifiers:*
*E = for some path*
*A = for all paths*




## Some CTL formulas

- I will like chocolate from now on, no matter what happens.
  AGp
- It is possible I may like chocolate some day, at least for one day.
  EFp
- No matter what happens now, In the future it is possible that I will suddenly start liking chocolate for the rest of time.
  AFEGp

## CTL: Exercises

Let p = 'I like chocolate' and q = 'It is warm outside'.
Write in CTL:

1. *I will never like chocolate.*
2. *From now until it's warm outside, I will like chocolate every single day (no matter what happens).*
3. ** It's possible that: there will eventually come a time when it will be warm forever, and that before that time there will always be some way to get me to like chocolate the next day.*

## Nice, this logic..

But what is it good for?

One application is *Formal Verification:*

*the use of mathematical techniques to ensure that a design conforms to some precisely expressed notion of functional correctness.*

## Formal Verification

System ⟶ A mathematical model M
Desired behavior ⟶ A formal specification ϕ
Correctness ⟶ Check whether M meets ϕ

Chris Langmead's presentation this afternoon!

## Temporal statements can be tricky

Suppose professor Griffeth tells you:

"There will be an exam (next week), but you will not know in advance which day."

What is strange about this statement?

## Temporal statements can be tricky

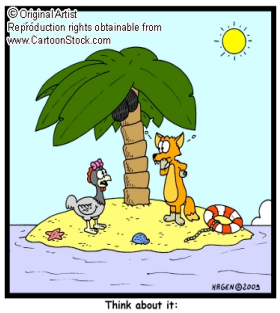
Well, the exam cannot be on Friday, since we would know it by Thursday night.

But the exam cannot be on Thursday either, since we would know this by Wednesday night (we eliminated Friday above, so the only possibility would be Thursday).

Similarly, we can eliminate the other days of the week... Therefore, we cannot have an exam next week.

So, surprise examinations do not exist!

## Questions?




## References

1. Yde Venema *Temporal Logic.* Guide to Philosophical Logic (2001*)*
2. Wikipedia (entries: logic, temporal logic, Linear temporal logic, CTL)
3. Stanford Encyclopedia (entries: time, temporal logic)
4. Cartoonstock.com