Correction to \textit{FOIL Axiomatized}
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There is an error in the completeness proof for the \{\lambda, =\} part of \textit{FOIL}-K. The error occurs in Section 4, in the text following the proof of Corollary 4.7, and concerns the definition of the interpretation $I$ on relation symbols. Before this point in the paper, for each object variable $v$ an equivalence class $v$ has been defined, and for each intension variable $f$ a function $\overline{f}$ has been defined. Then the following definition is given for a relation symbol $P$: 
$\langle v_1, v_2, \ldots, f_1, f_2, \ldots \rangle \in I(P)(\Gamma)$ just in case there are $w_1, w_2, \ldots$ in $d(\Gamma)$ with $w_i \in \overline{v_i}$ such that $P(w_1, w_2, \ldots, f_1, f_2, \ldots) \in \Gamma$. It was pointed out by Torben Brauner that we could have $f_1$ and $g_1$ being the same function, but also have $P(w_1, w_2, \ldots, f_1, f_2, \ldots) \in \Gamma$ without having $P(w_1, w_2, \ldots, g_1, f_2, \ldots) \in \Gamma$.

Our solution is to modify the definition of the model, rather artificially, so that if $f$ and $g$ are the same function, then $f$ and $g$ are syntactically the same intension variable. This is done as follows. First, arbitrarily choose some object variable $w$, and its corresponding equivalence class $\overline{w}$. For each intension variable $f$ we define a disambiguation world $\hat{f}$ as follows. Technically $\hat{f}$ must be some entity—it will not matter what we choose, pick any entity for this. We simply need that for distinct $f$ and $g$ we have $\hat{f} \neq \hat{g}$. For each intension variable $g$ other than $f$, extend $\overline{g}$ so that $\hat{f}$ is in its domain, and at this world $\overline{g}$ has the value $\overline{w}$. For $f$ itself, the world $\hat{f}$ is not in the domain of $\overline{f}$.

Modify the definition of the model $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}_O, \mathcal{D}_I, \mathcal{I} \rangle$ as follows. $\mathcal{G}$ is enlarged to include all disambiguation worlds, $\hat{f}$, as well as the members given to it in the paper. Call the members of $\mathcal{G}$ that are not disambiguation worlds, that is, members assigned to $\mathcal{G}$ in the paper, standard worlds. $\mathcal{R}$ and $\mathcal{D}_O$ are not changed. $\mathcal{D}_I$ is still to be all $\overline{f}$ for intension variables $f$, but the partial function $\overline{f}$ will now have disambiguation worlds other than $\hat{f}$ in its domain. $\mathcal{I}$ is formally as before.

In the modified model, if $f$ and $g$ are different intension variables, $\overline{f}$ and $\overline{g}$ will be different functions, because $\hat{f}$ will be in the domain of $\overline{g}$ but not in the domain of $\overline{f}$. Now the definition of $\mathcal{I}$ on relation symbols is no longer problematic. Finally the Truth Lemma, Proposition 4.8, and its proof, must be modified so that the results are only claimed for standard worlds $\Gamma$. 