

Title Normal surface theory and the colored Jones polynomial

Abstract The colored Jones polynomial is a generalization of the Jones polynomial from the finite-dimensional representations of $U_q(\mathfrak{sl}_2)$. One motivating question in quantum topology is to understand how the polynomial relates to other knot invariants. An interesting example is the strong slope conjecture, which relates the asymptotics of the degree of the polynomial to the slopes of essential surfaces of a knot. Motivated by the recent progress on the conjecture, we discuss a connection from the colored Jones polynomial of a knot to the normal surface theory of its complement. We give a map relating generators of a state-sum expansion of the polynomial to normal subsets of a triangulation of the knot complement. Besides direct applications to the slope conjecture, we will also discuss potential applications to colored Khovanov homology.