Non-abelian covers of knots from the Alexander polynomial

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Abstract

In 1967 de Rham simultaneously gave an alternate definition of the Alexander polynomial and constructed complex affine homomorphisms of $\pi_1\left(\mathbf{S}^3 \setminus K\right)$, for K any tame knot embedded in $\mathbf{S}^3$. In this talk we analyze the analogue of de Rham's construction modulo a prime p. This allows us to define explicit, non-trivial $\rho : \pi_1\left(\mathbf{S}^3 \setminus K\right) \to \mathrm{GL}(2, \mathbb{F}_{p^d})$, when K has non-trivial Alexander polynomial. We then turn our attention to the cover X_ρ corresponding to $\ker(\rho)$, this cover can be used to describe upper bounds on the index of non-cyclic covers of $\mathbf{S}^3 \setminus K$, similar to the results of Broaddus and Kuperberg. Lastly, time permitting, we discuss $H_1\left(X_\rho\right)$ establishing bounds for the first betti number, effectively showing that the virtual betti number of such knot complements is infinite, which is an alternate proof of the famous result due to Cooper, Long, and Reid.