3–Manifolds, Artin Groups and Cubical Geometry.

Problems for the first recitation, Tuesday 2 August. Please email questions or corrections to pprzytyc@mimuw.edu.pl. Solutions partially in Dani Wise's lecture notes.

Exercise 1. Prove that the Dehn complex of a projection of a link is nonpositively curved if and only if the link projection is alternating and prime.

Exercise 2 (Bigon removal)**.** Let X be a nonpositively curved cube complex and let $D \to X$ be a disc diagram. Assume that D has minimal area among all disc diagrams with common boundary. Prove that the dual curves of the diagram cannot form a bigon or a monogon.

Exercise 3. Let X be a special cube complex. Prove that X admits a local isometry into the Salvetti complex (i.e. the presentation complex) of a right angled Artin group.

Exercise 4. Let $Y \to X$ be the following local isometry of cube complexes. Compute the canonical completion and retraction of that map.

(1) X is a rose with two petals a and b. Y is the immersed loop of length 4: $aba^{-1}b^{-1}$.
(2) see Figure 1.

Exercise 5 (* Dani Wise's PhD example)**.** Let X be the square complex glued out of the six squares in Figure 2.

(1) Let Π denote the flat plane quadrant spanned by the horizontal double arrow halfline going right and the vertical black halfline going up. Prove that Π is not periodic.
(2) Let $\widetilde{X}$ be any finite cover of X and let $\widetilde{V}$ be the union of the vertical edges of $\widetilde{X}$. Let Γ be the graph whose vertices are connected components of $\widetilde{V}$ and whose edges are connected components of the complement of $\widetilde{V}$ in $\widetilde{X}$. Prove that Γ is a cycle.
(3) Deduce that $\pi_1(X)$ is not $\pi_1(V)$ separable. Prove that the fundamental group of the double of X along V is not residually finite.

Figure 1

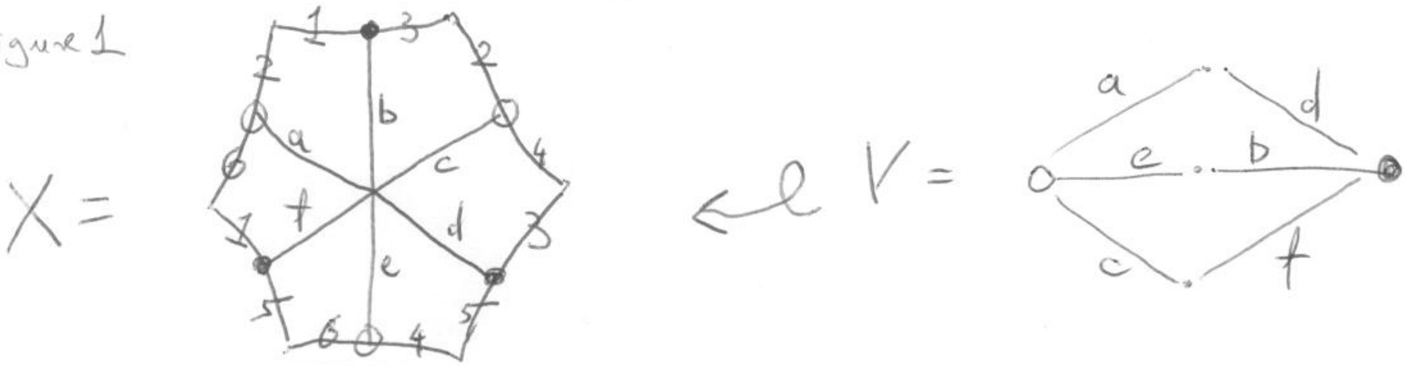

Figure 2